Zero Knowledge Proofs for Constructing Protocols

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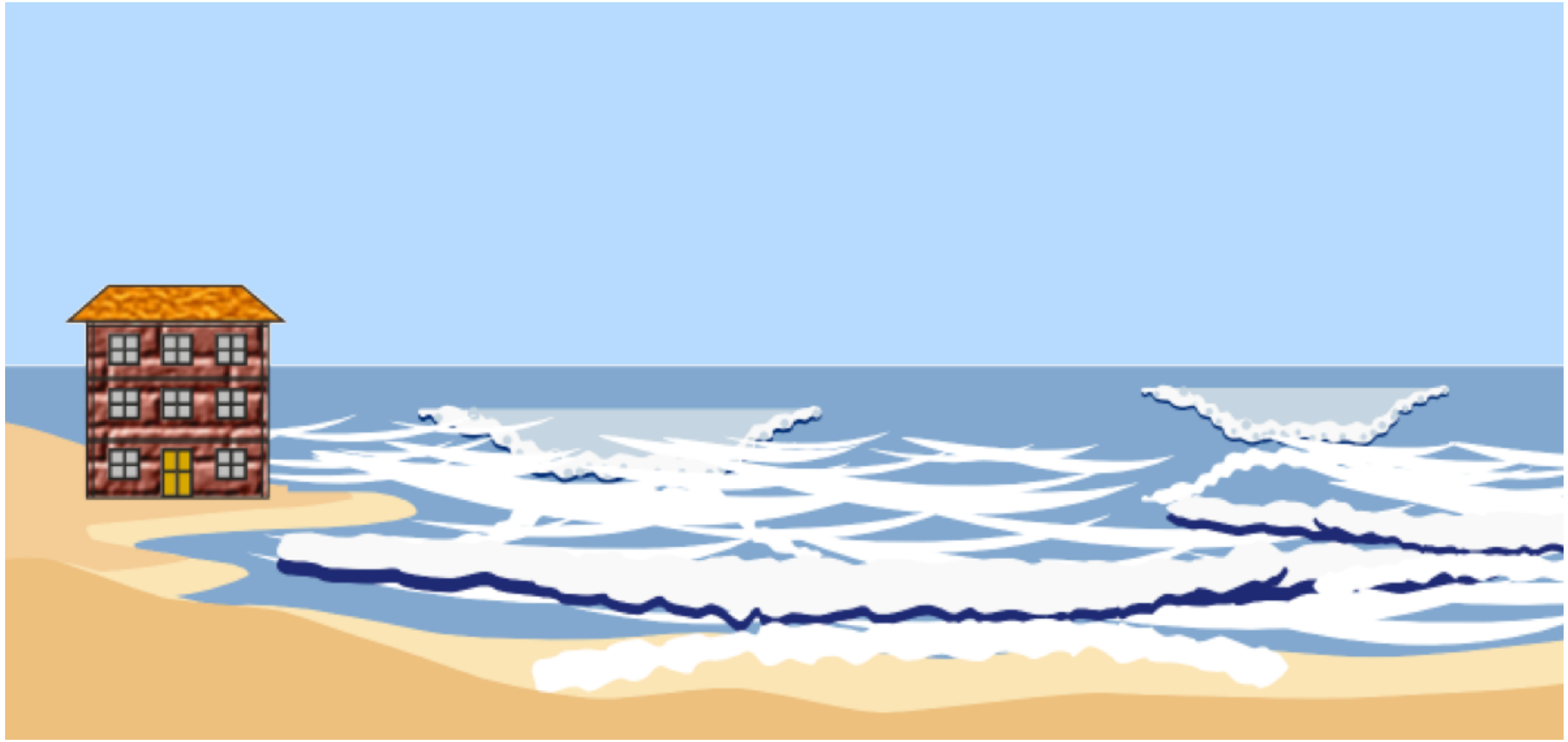
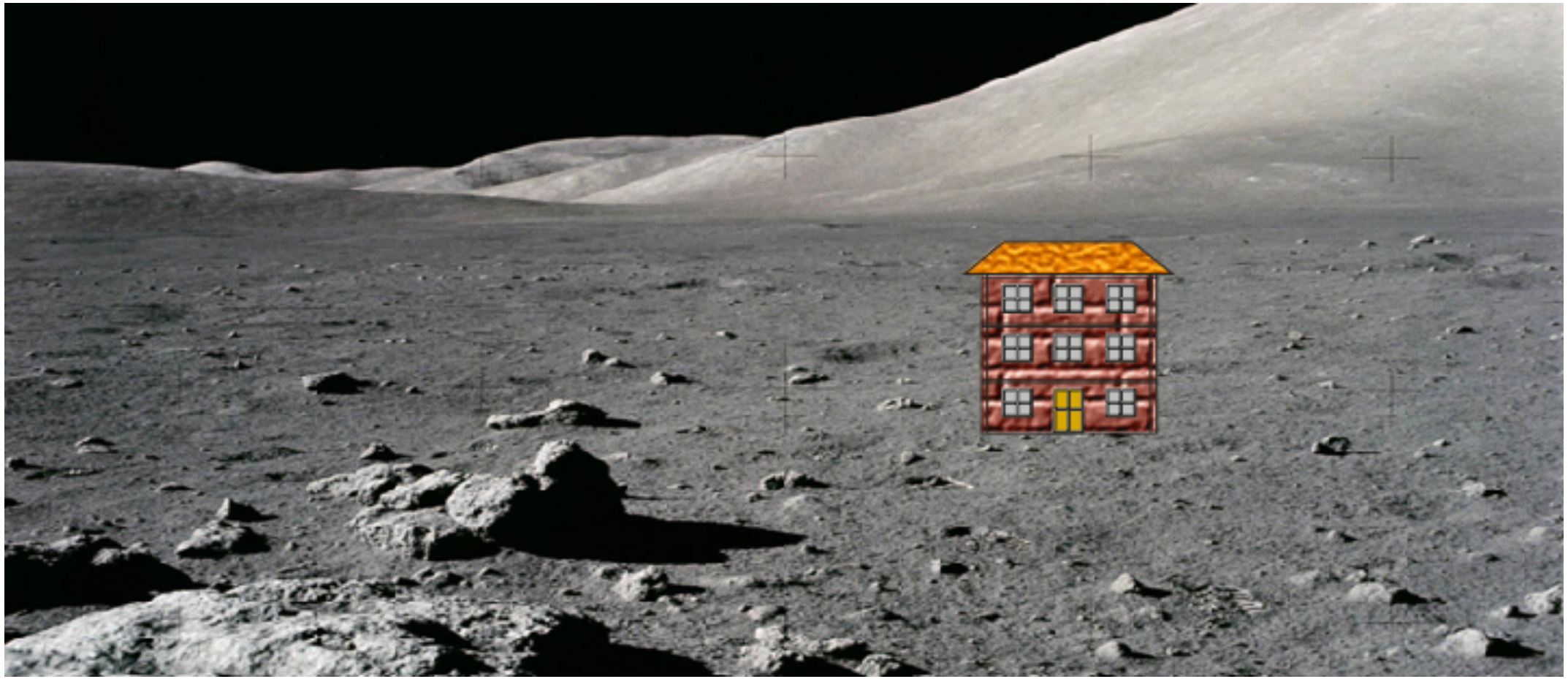
DFINITY

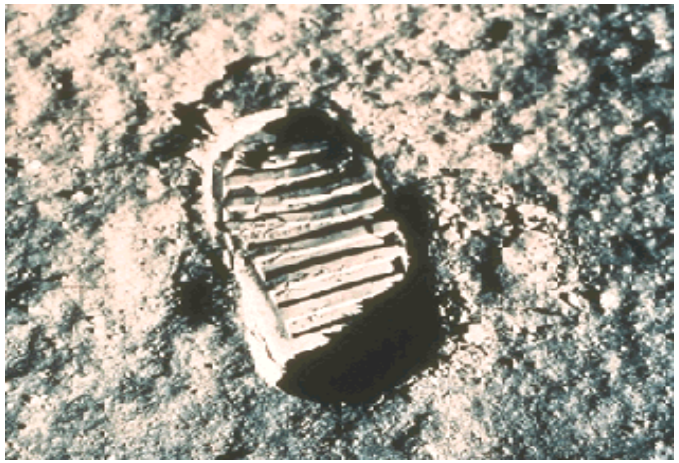
Cryptography is a constructive science



… but end up doing this



Computers never forget

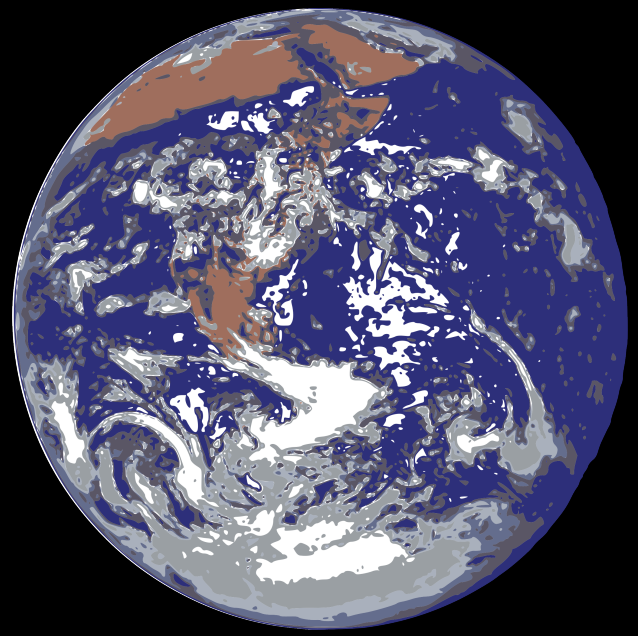


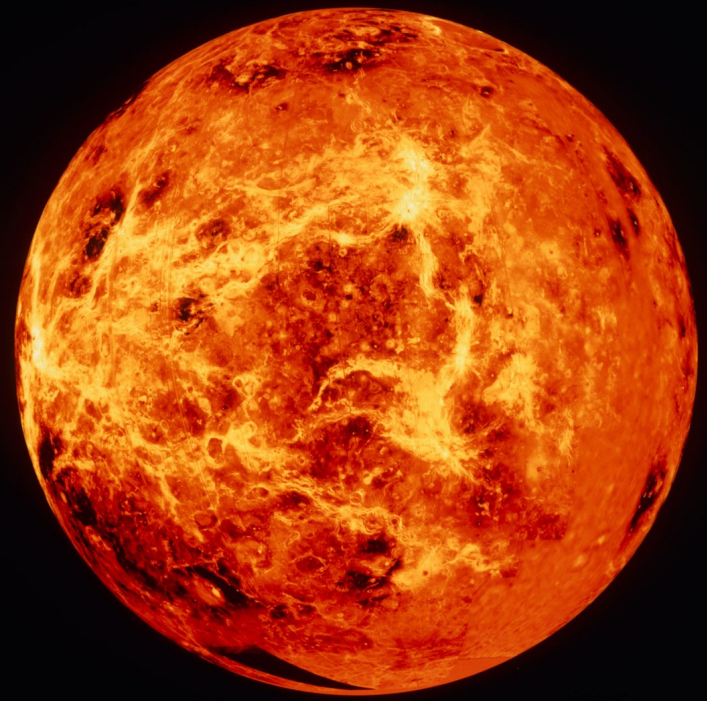
- Data is stored by default
- Data mining gets ever better
- Apps built to use & generate (too much) data
- New (ways of) businesses using personal data

- Humans forget most things too quickly
- Paper collects dust in drawers

But that's how we design and build applications!

Apps securely deployable in any environments





Building & proving secure a cryptographic protocol is hard

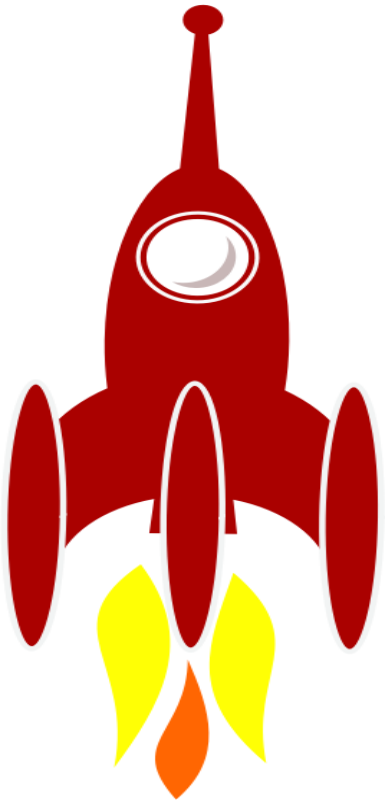
- Finding the right abstraction
- Building an *efficient* realisation modularly from primitives
- Prove security of realisation
- Instantiate primitives to obtain protocol

Cryptography is one of the main ingredients to space faring!

Let’s talk about:

Zero knowledge proofs

in space faring

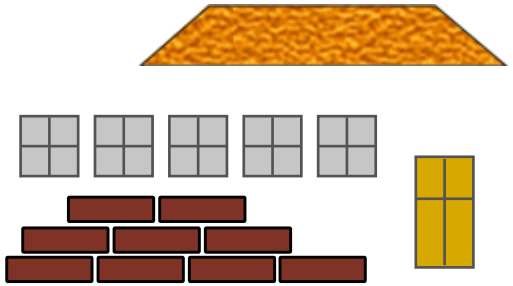


Part I — Practical aspect

Zero-knowledge proofs and other protocol building blocks

Well known and well defined primitives as building blocks

- Signature Schemes
- Encryption Schemes
- Commitment Schemes
- Verifiable Pseudo Random Functions
- ….



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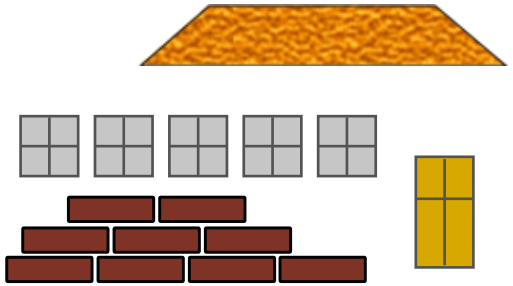
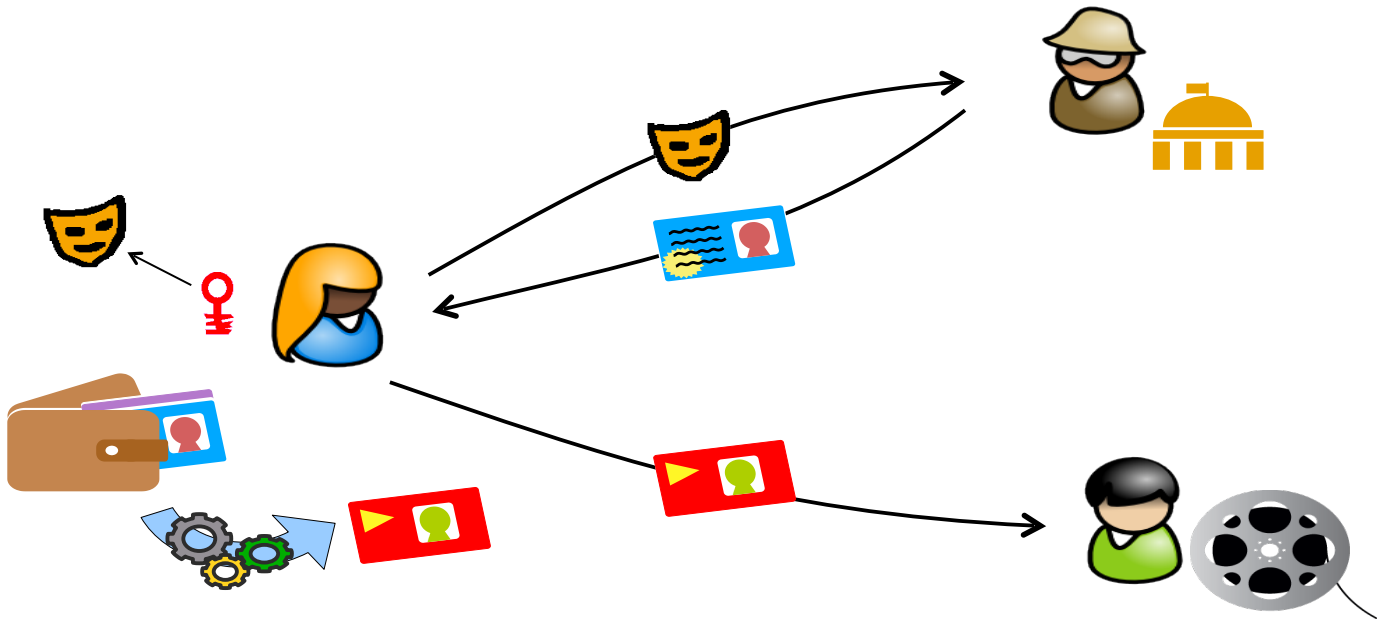


..and, of course, Zero-Knowledge Proofs as mortar!

Works well in theory, but rather tricky for practical protocols

Example application: privacy-protecting authentication



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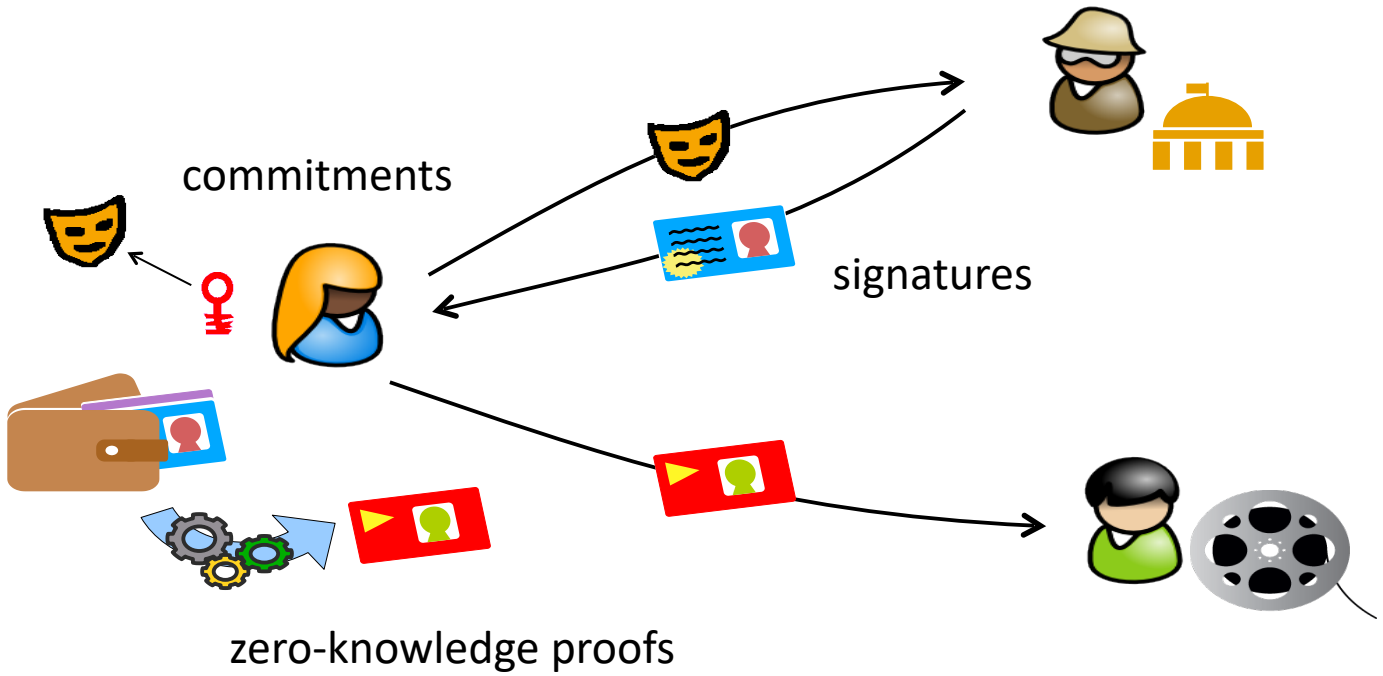
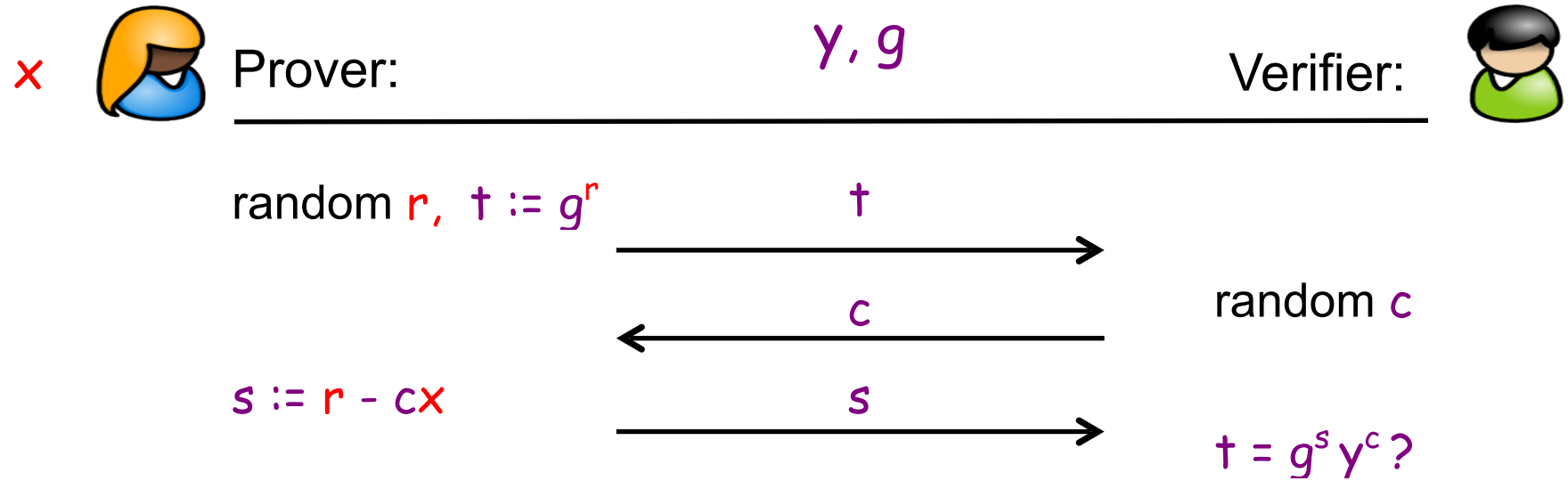
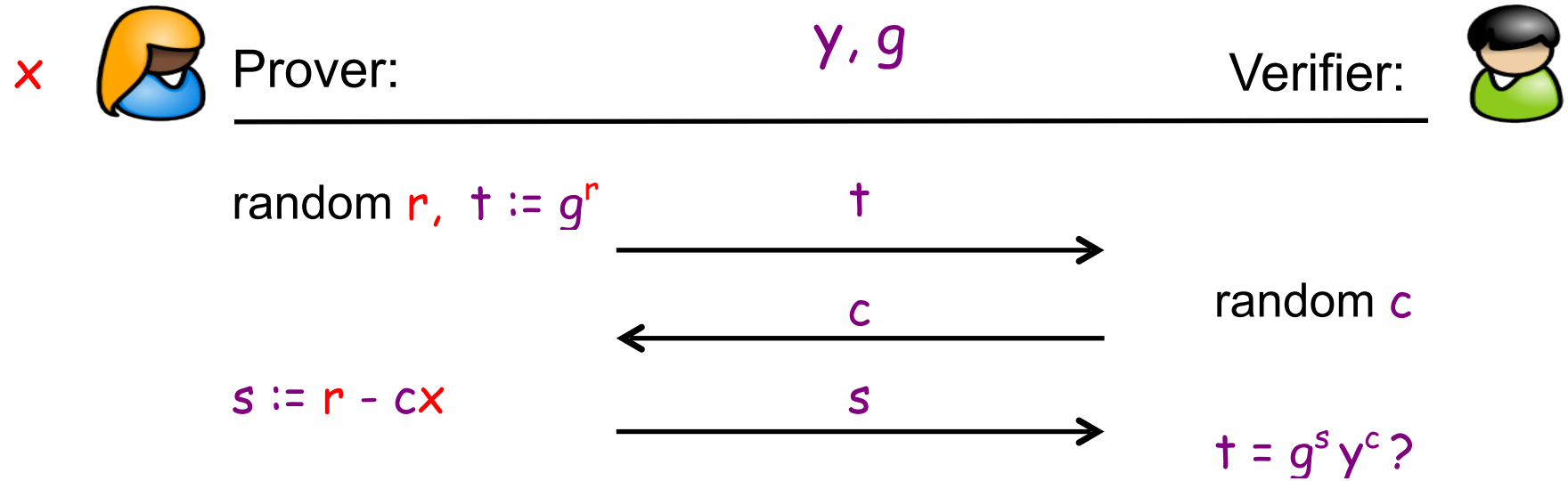


Efficient Zero-Knowledge Proofs

The well known Schnorr protocol



Efficient Zero-Knowledge Proofs



Proof of knowledge: from a successful prover, *using rewinding*, one can extract x s.t. $y = g^x$

thus notation: PK{(x): $y = g^x$ }

Zero knowledge: the verifier learns nothing from interacting in protocol with prover

Non-interactive with Fiat-Shamir heuristic, i.e., $c = H(t,y,g,m)$. -> SPK{(x): $y = g^x$ }(m)

Efficient Zero-Knowledge Proofs

Important observations:

1. Protocol works for any group homomorphism f: $G_1 \to G$
2. Prover and verifier have work equivalent to one evaluation of f

Prover:

$y = f(x)$

Verifier:

random r, $t=f(r)$

t

random c

c

$s := r - cx$

s

$t = f(s)\, y^c$?

Efficient Zero-Knowledge Proofs

Protocol works for *any group homomorphism* f: $G_1 \to G_2$

- Classical examples

  $x \to g^x$ $\quad$ $(u,v) \to g^u h^v$ $\quad$ $(u,v) \to (k^v, g^u h^v, g^v)$

- Bilinear maps e: $G_1 \times G_1 \to G_t$

  $x \to e(g,g)^x$ $\quad$ $(u,v) \to e(g, g^u h^v)$ $\quad$ $z \to e(z,g)$

- But not

  $(u,v) \to g^{u\,v}$ $\quad$ $(u,v) \to e(k^v, g^u h^v)$ or $(x,z) \to e(z,g^x)$

Logical operators work as well:

- PK{(x,u,v,z): $a = g^x \wedge b = e(z,g) \wedge d = g^u h^v$}
- PK{(x,u,v,z): $a = g^x \vee (b = e(z,g) \wedge d = g^u h^v)$}

Efficient Zero-Knowledge Proofs

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Can be standardised! :-)

Well known and well defined primitives as building blocks

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- Commitment Schemes
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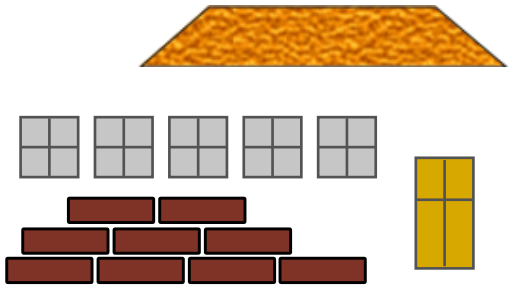
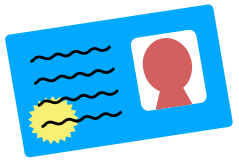
- …



… that are compatible, e.g. signature s.t. PK{(s,m): 1 = verify_signature(PK,s,m)}

i.e., 1 = verify_signature(PK,s,m) is in the language of the proof

Example signature scheme



Goal: Want to have PK{(s,m): 1 = verify_signature(PK,s,m)}

Take, e.g., Pointcheval-Sanders signature scheme:

bilinear map setting $e: G_1 \times G_1 \rightarrow G_t$

secret key x, y public key $X=g^x$, $Y=g^y$

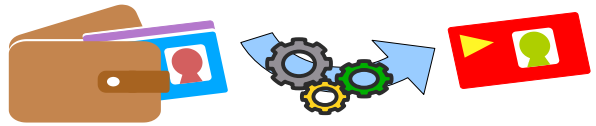
signature on a message m is tuple (a,b) s.t. $e(a,XY^m) = e(b,g)$

- with a random from G_1
- $b=a^{x+my}$

Leading to PK{(a,b,m): $1 = e(a,XY^m)\, e(b,g)^{-1}$ } …however, this is not a valid statement :(

Proof of knowledge of message and signature

So: PK{(a,b,m): $e(a,XY^m) = e(b,g)$ } which, however, is not a valid statement :(



Trick:

1. Let $(A,A') = (ag^r,h^r)$ i.e., encrypt a under

   -> $e(A,XY^m) = e(ag^r,XY^m) = e(b,g)\,e(g^r,XY^m) = e(b,g)\,e(g,XY^{rm})$

   -> $e(A,X) = e(b,g)\,e(g,XY^{rm})\,e(A,Y^{-m})$

2. Let $u = rm$ -> $e(A,X) = e(b,g)\,e(g,XY^u)\,e(A,Y^{-m}) \wedge A' = h^r \wedge 1 = A'^{-m}h^u$

Now we get

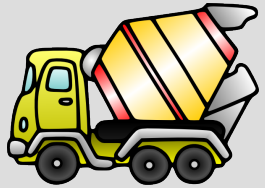
PK{(u,r,b,m): $e(A,X) = e(b,g)\,e(g,XY^u)\,e(A,Y^{-m}) \wedge A' = h^r \wedge 1 = A'^{-m}h^u$ }

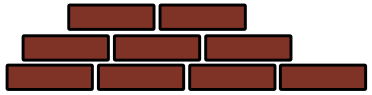
which is a valid statement and proves that (Ag^{-r},b) is a valid signature on m :)

Generalized Schnorr Proof Toolbox

Signature

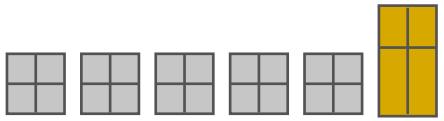
Camenisch-Lysyanskaya (RSA, Pairings)
Pointcheval-Sanders (Pairings)
BBS+ Signatures (Pairings)





Commitment

Pedersen
ElGamal



Encryption

Camenisch-Shoup
Camenisch-Damgaard
ElGamal

Alternative construction frameworks

Schnorr-proofs & discrete logarithms based primitives

+ well understood, lots of primitives, most efficient solutions
- non-interactive proofs under Fiat-Shamir heuristic

Groth-Sahai proofs & structure-preserving primitives

+ well understood, fair amount of primitives, requires pairings (less efficient), short proofs
- security for many primitives rely on generic group model

SNARKS, STARKS, bullet proofs & (algebraic) circuits

+ very expressive in terms of statements, short proofs
- much still in progress, need to design & research more primitives

Lattice-based proofs & primitives

+ quantum safe, STARKS like proofs can be applied
- some non-interactive proofs require Fiat-Shamir heuristic
- very early stages, true quantum security ages away

Part II — Theoretical aspect

Difficulties in proving ZKP-using protocols UC-secure

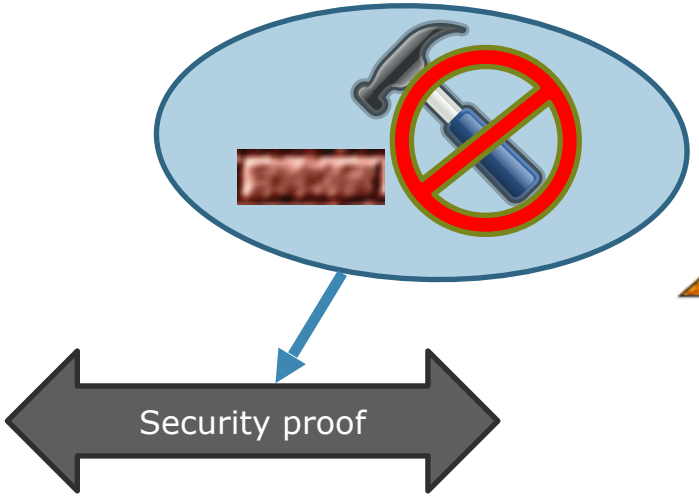
Cryptographic Protocol Design & Proofs

Specification:
functional and
security properties

Hybrid protocol

Actual implementation

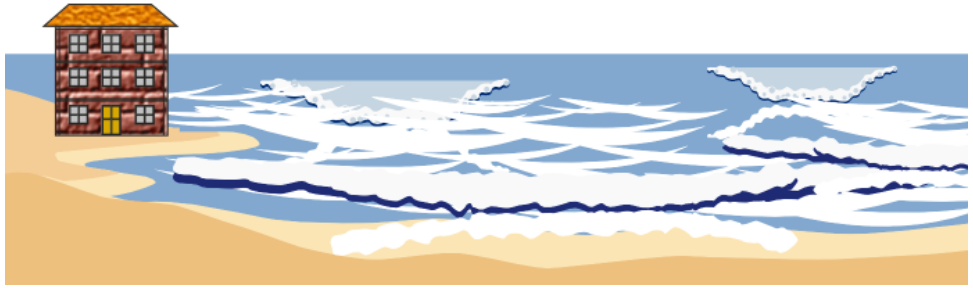
Cryptographic Protocol Design & Proofs: Goal

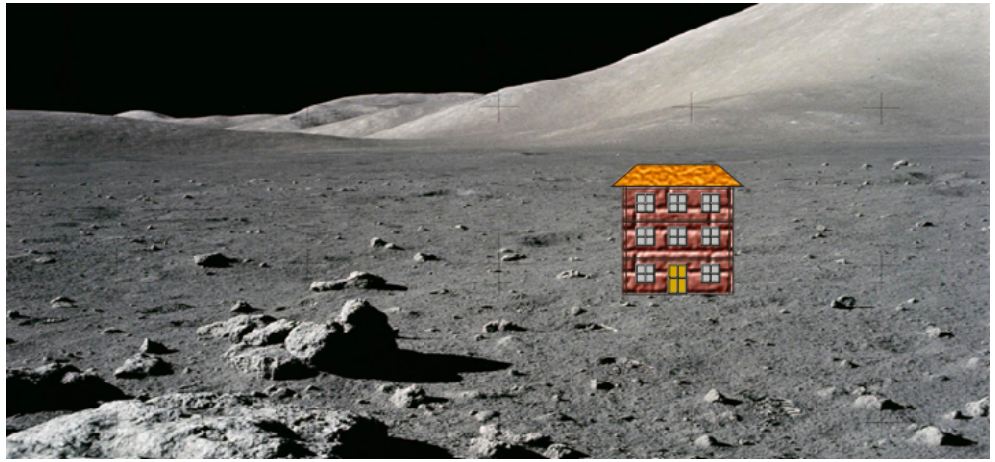


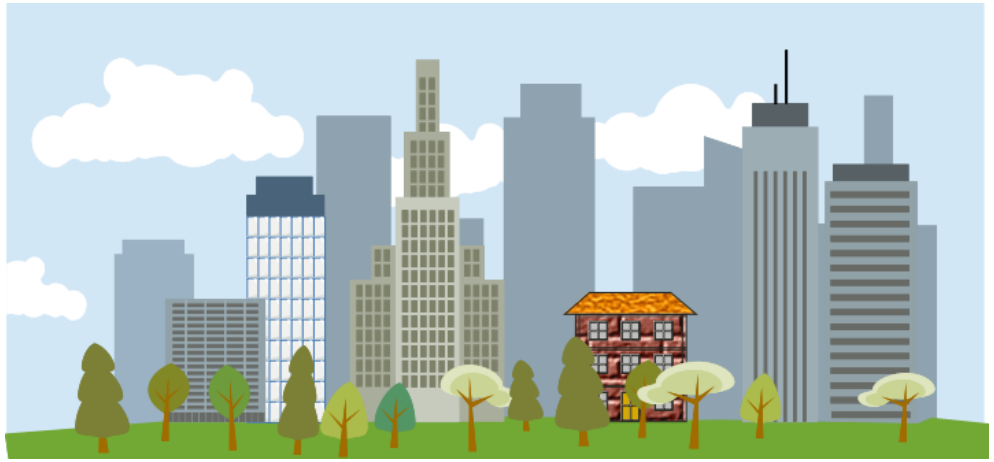
Hybrid Protocol

Actual implementation

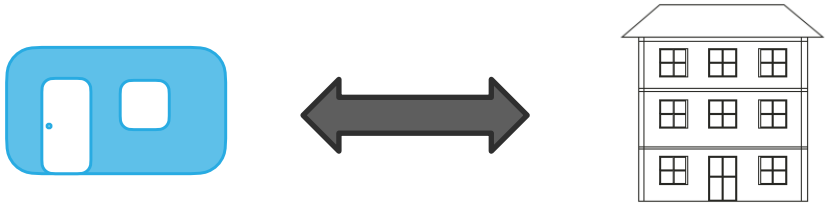
Wanna guarantee security in *any* environment







Modular Cryptographic Protocol Design



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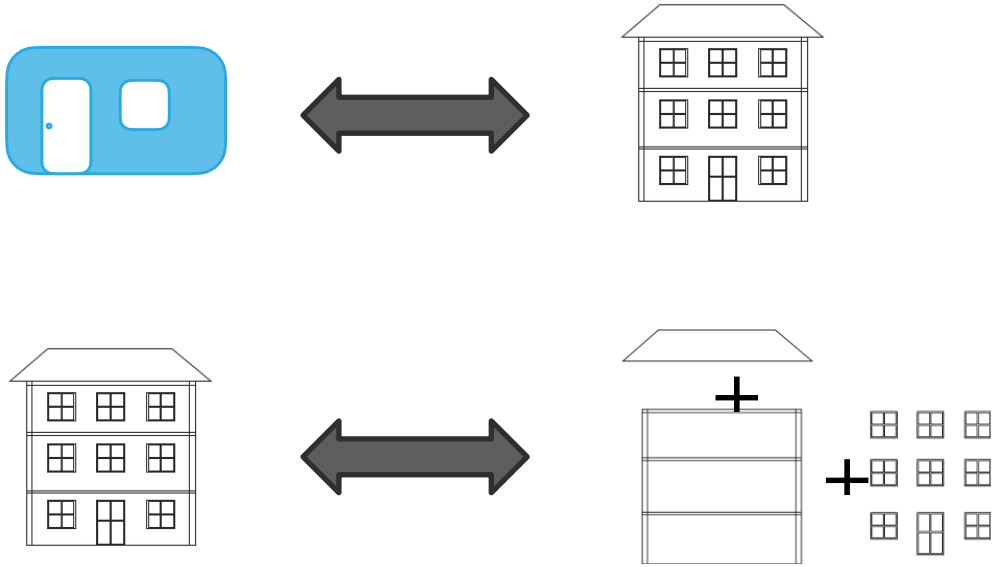

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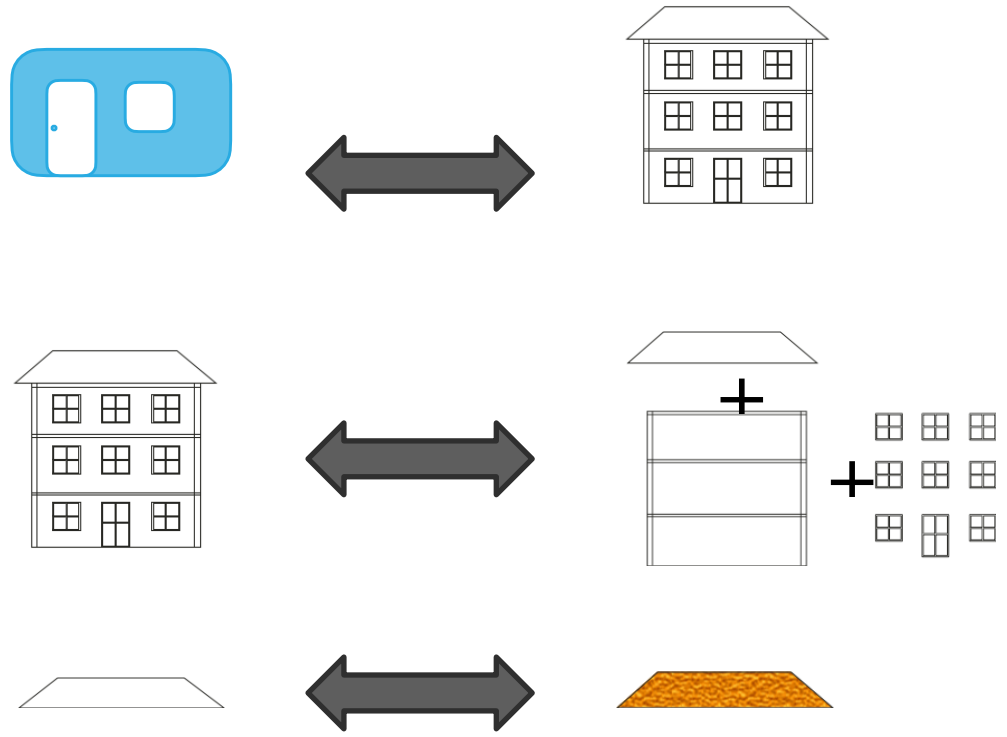

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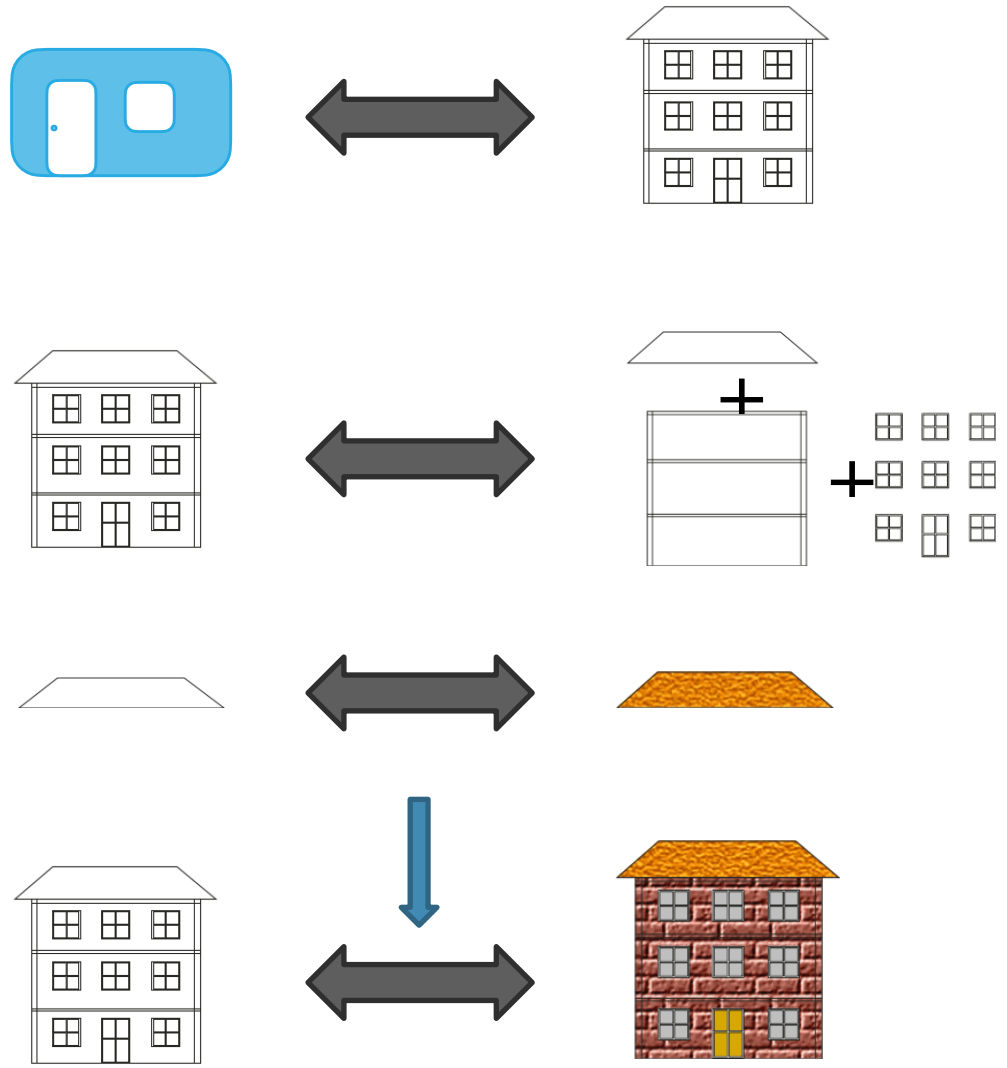

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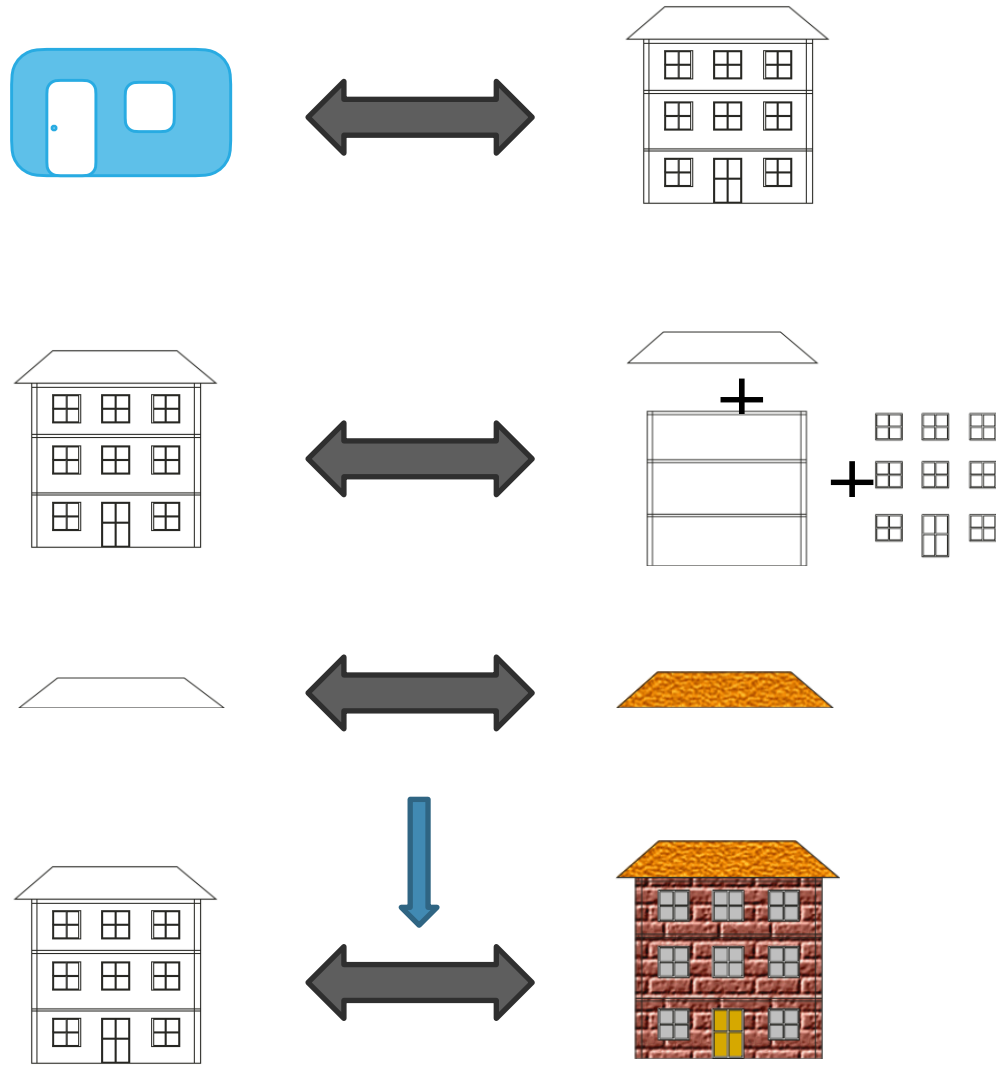
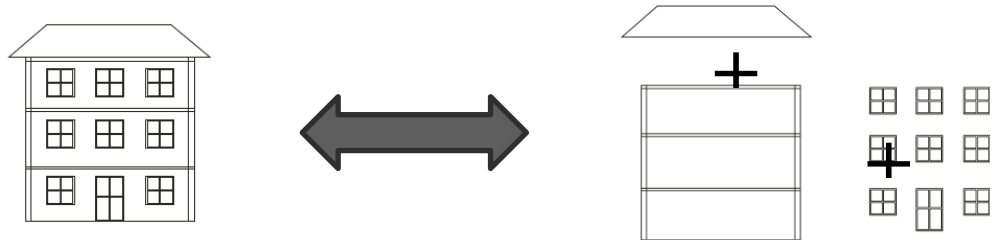


Needs to be done each time
Should be sufficient!

Library of secure primitives

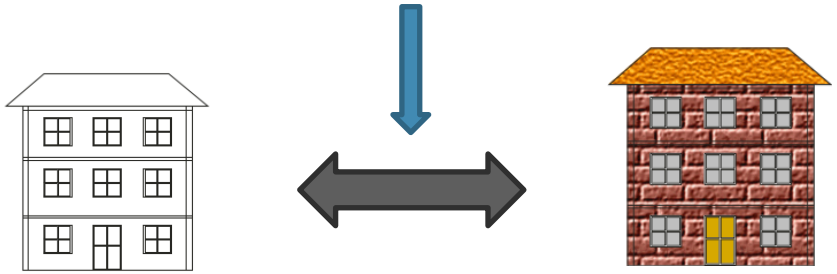
Follows from security framework

State of the art & challenges



Define building blocks that can be
- combined suitably & efficiently
- allow for modular proofs

Provide efficient and secure realizations of building blocks



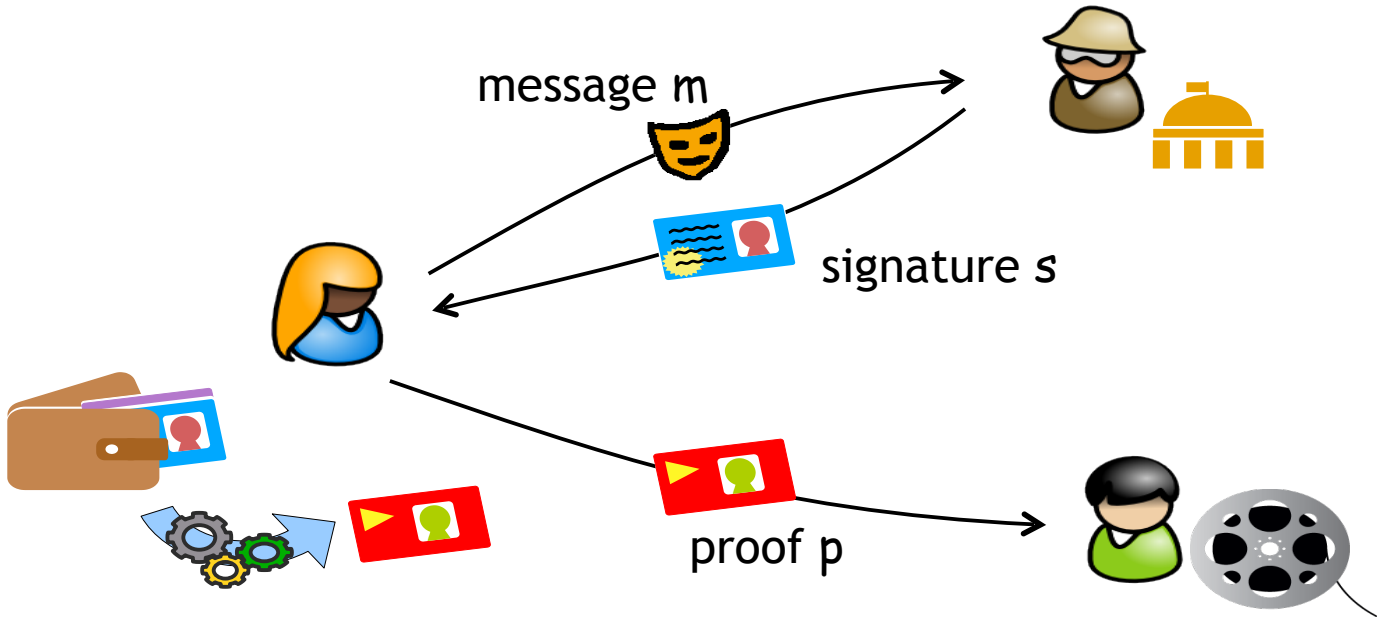
a) Security Composition Frameworks (UC et al.)
- hardly ever used like this
- definitions of building blocks still requires research!

b) Automatic with set of property-based definitions
- typically how people do it
- very involved (and first step sometime not sound)

Goals of (security) composition frameworks

- Modular construction with modular proof!
  - Functionalities are idealized
  - Protocol uses functionalities in a modular way
  - As protocol is modular, proof is be modular and, in particular, abstract and simple
  - As functionalities model the security of the schemes, no additional reduction to any property of the schemes should be necessary!

Consider (very) simple anonymous credential use case

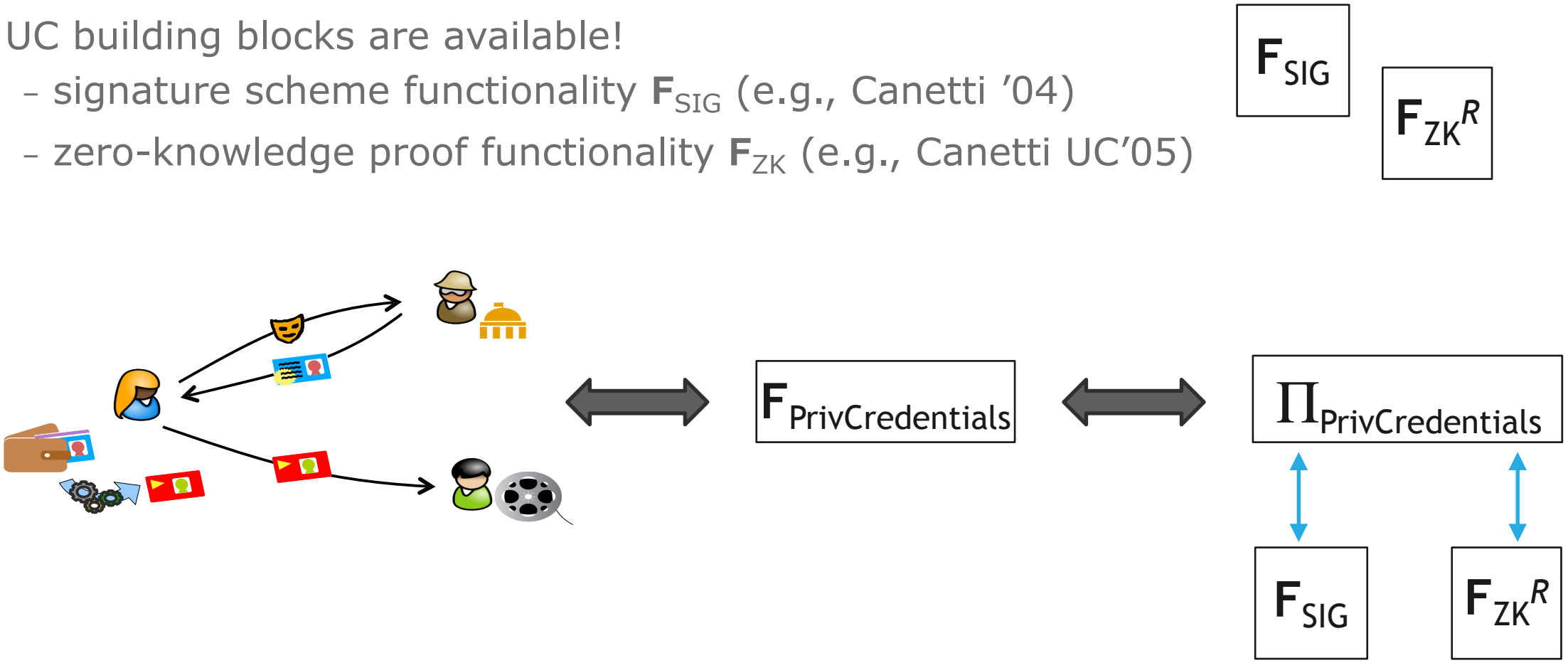


p = PK{(s,m): 1 = verify_signature(PK,s,m)}

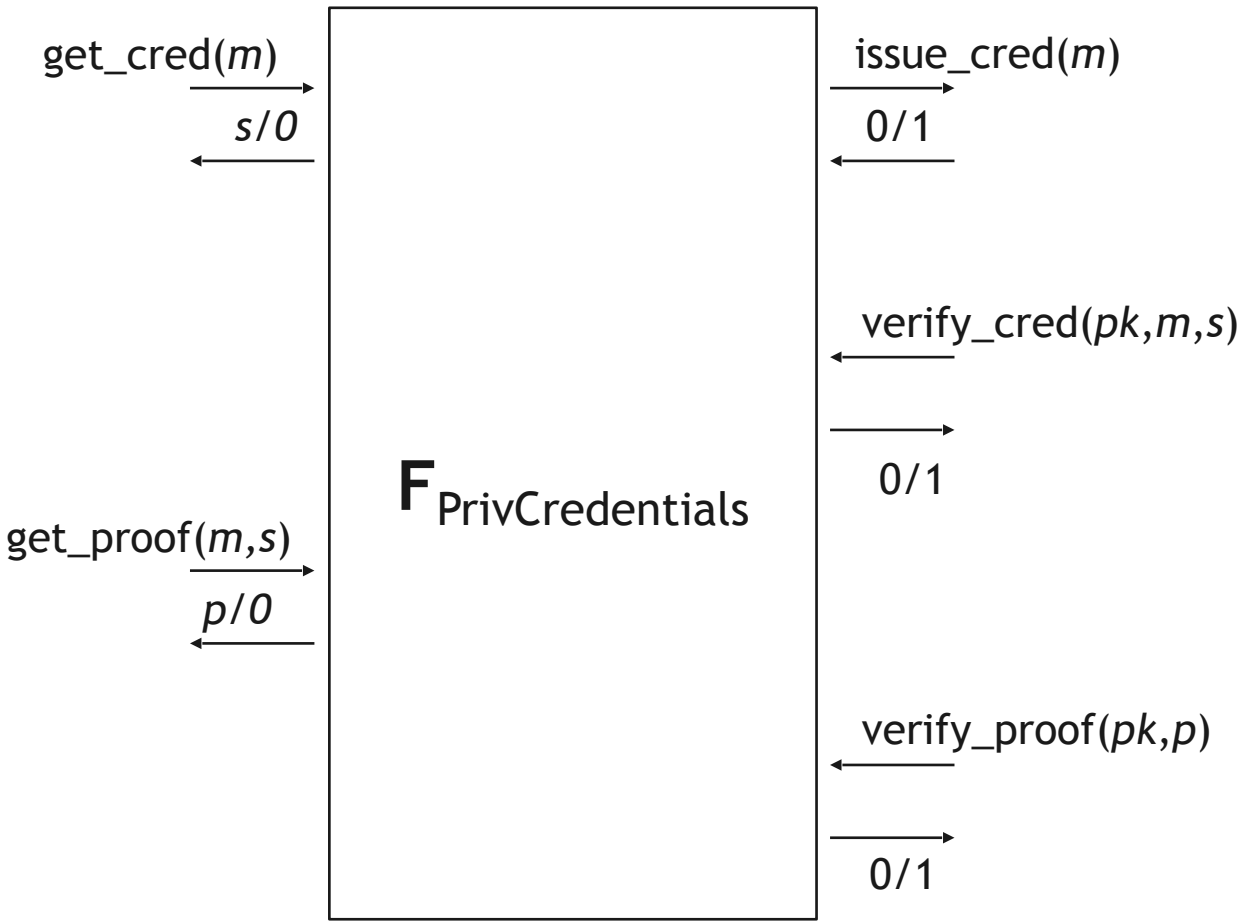
Let us UC-dream…. modular protocol construction

UC building blocks are available!

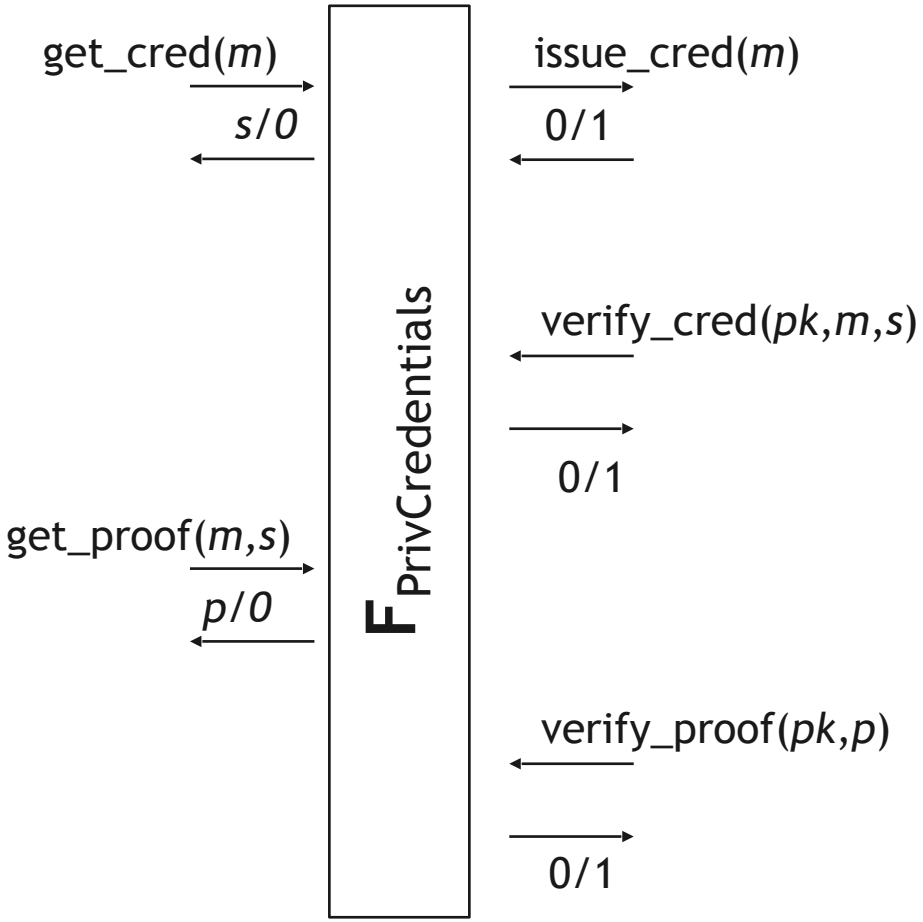
- signature scheme functionality $\mathbf{F}_{SIG}$ (e.g., Canetti '04)
- zero-knowledge proof functionality $\mathbf{F}_{ZK}$ (e.g., Canetti UC'05)

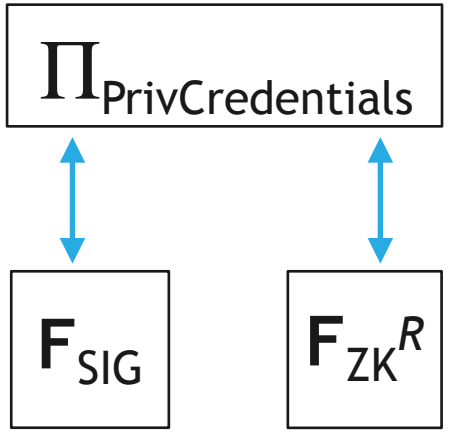


Let us UC-dream…. modular protocol construction

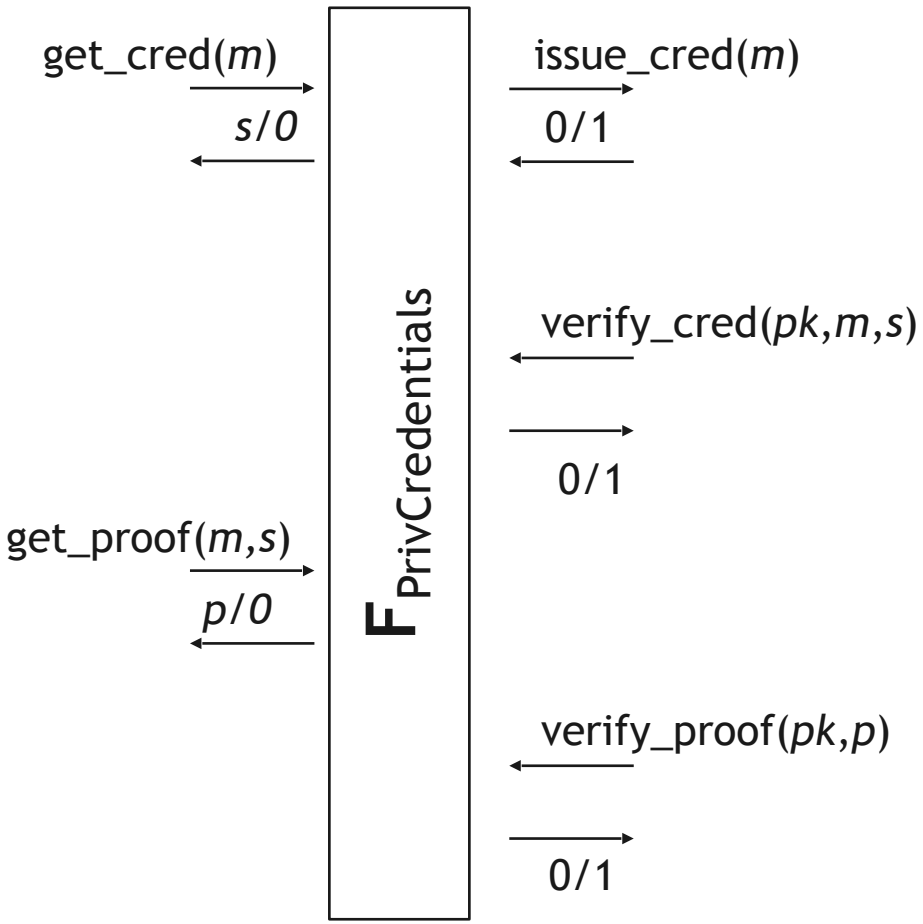


Let us UC-dream…. modular protocol construction



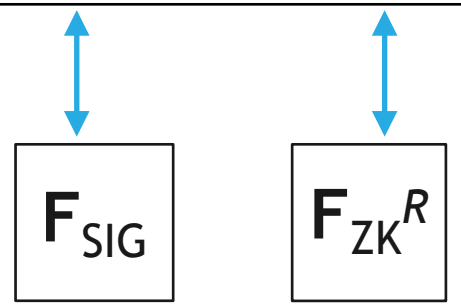


Let us UC-dream…. modular protocol construction

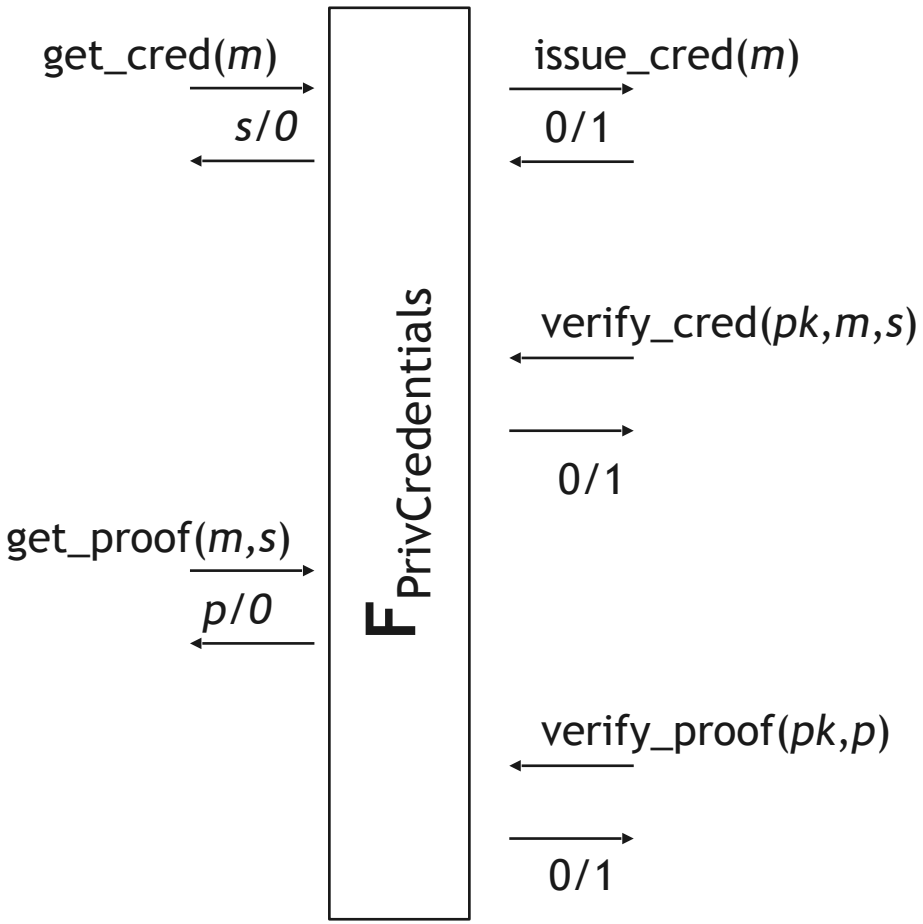


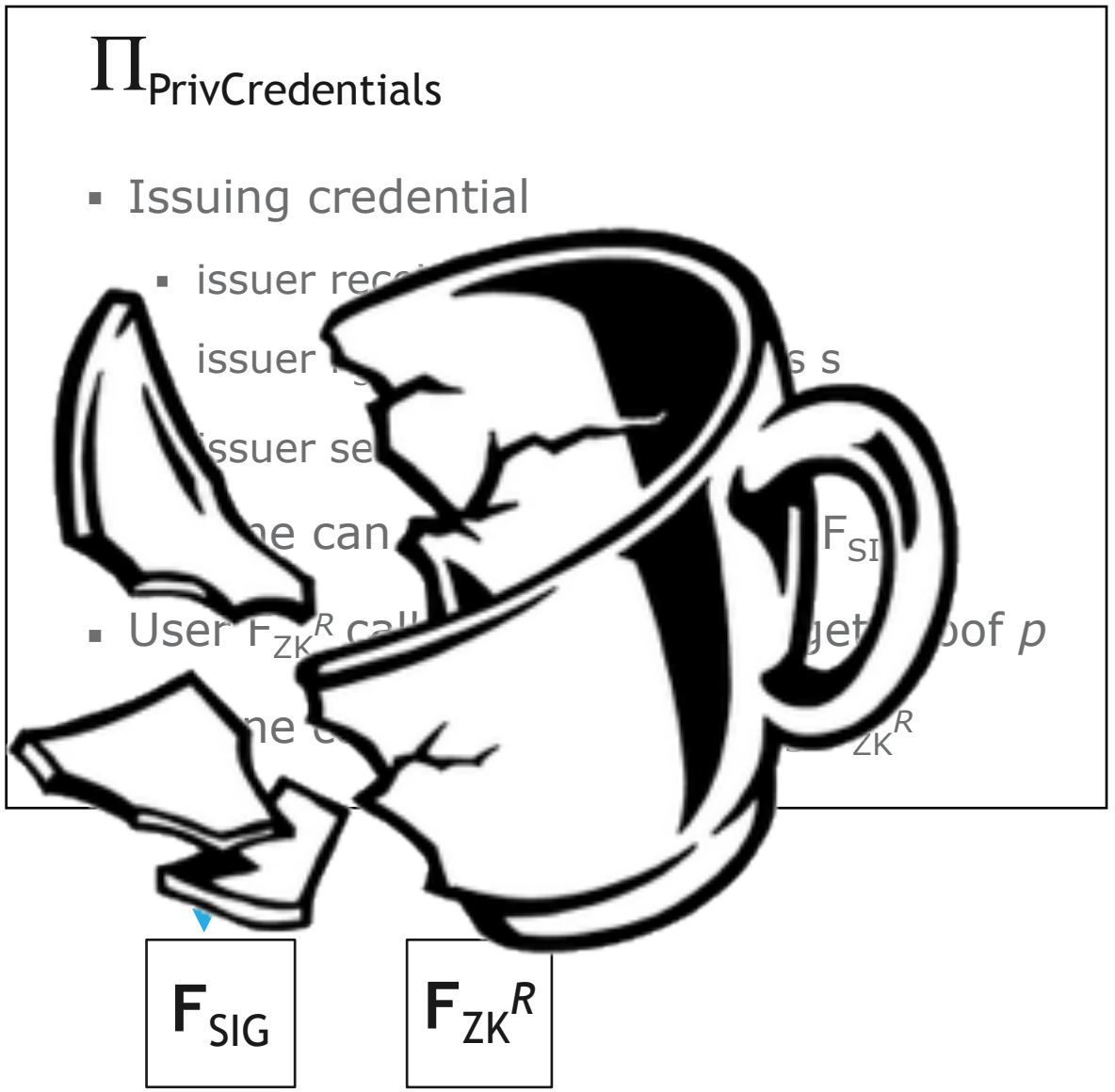
$\Pi_{\text{PrivCredentials}}$

- Issuing credential
  - issuer receive *m* from user
  - issuer F_{SIG} calls on *m* and gets s
  - issuer send *s* to user
- Anyone can verify s by calling F_{SIG}
- User F_{ZK}^{R} calls on *m, s, pk* to get proof *p*
- Anyone can verify *p* by calling F_{ZK}^{R}

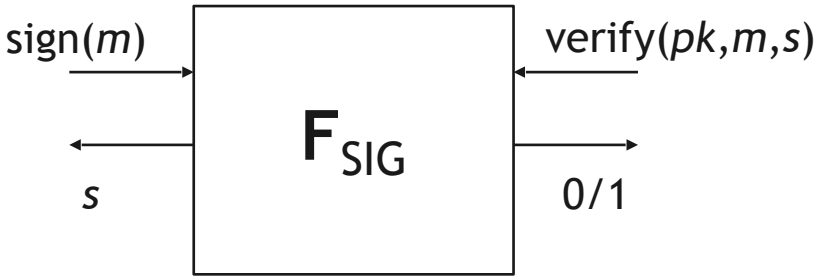


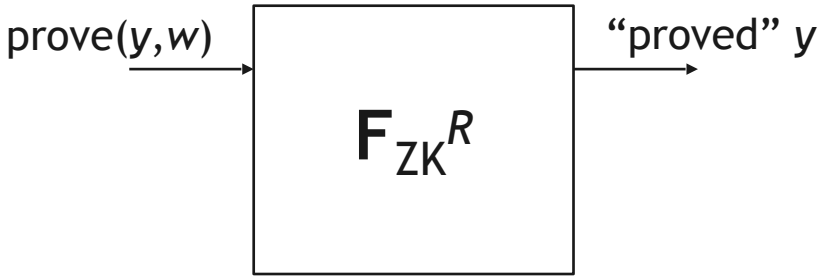
Let us UC-dream…. modular protocol construction





Let us investigate our building blocks





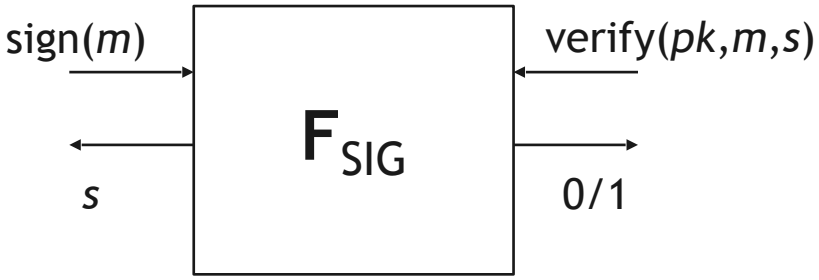
Different versions of F_{SIG} exist:

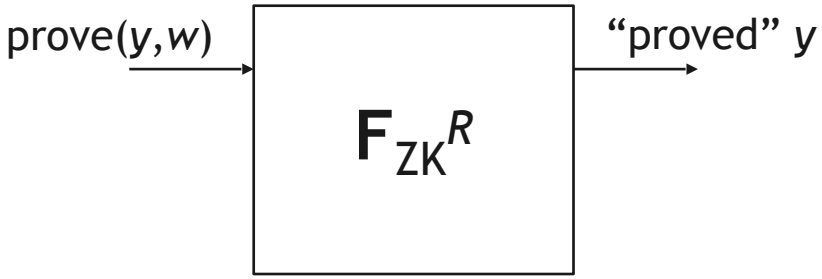
- Every signature generated by adversary (e.g., Canetti '04)
- Adversary initially uploads algorithms (e.g., Canetti UC'05, Küsters-Tuengerthal '13)

The relation *R* is between statement *y* and witness *w* and parametrizes the functionality. The proof is of course only accepted if (*y*,*w*) is in the relation *R*, and the functionality models a proof of knowledge.

(*y*, *w*) in *R* if *y* = (*pk*, *crs*) and *w* = (*m*, *s*) and verify(*pk*, *m*, *s*) = 1

Let us investigate our building blocks





A number of stumbling stones:

- F_{ZK}^{R} is interactive, but we need to get a "proof" string
- Relation *R* talks about a specific algorithm, i.e., *verify(pk,m,s) =1*, but F_{SIG} is agnostic of its algorithms, i.e., defined for *any* algorithms
- The statement *verify(pk,m,s) =1* does not imply that F_{SIG} would answer with 1

Towards removing the stumbling stones

A number of stumbling stones:

- F_{ZK}^{R} is interactive, but need to get a "proof" string:
  - Use (modification of) F_{NIZK}^{R} that outputs a proof [Groth et al 2012]

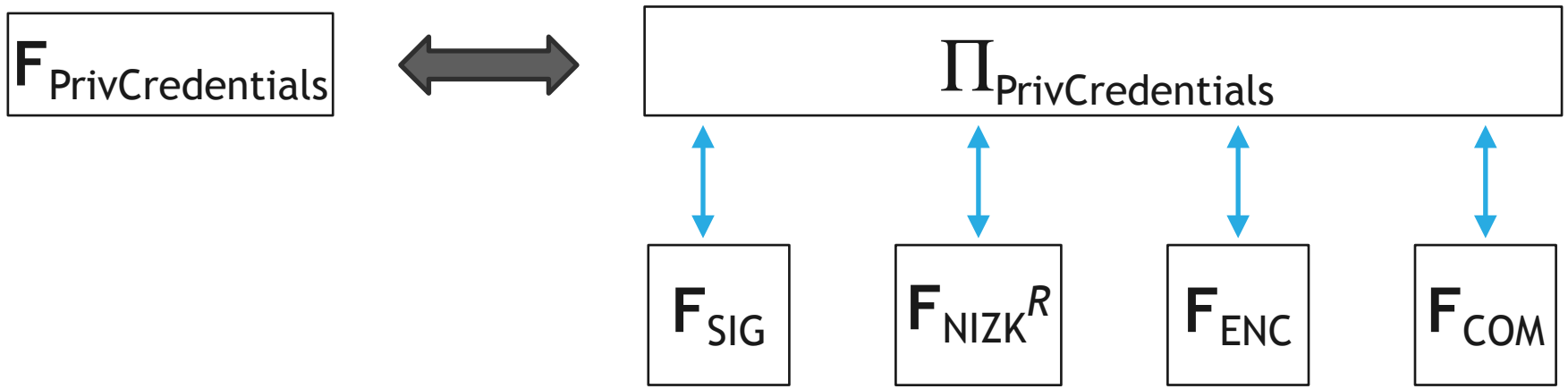
- Relation *R* talks about a specific algorithm, i.e., *verify(pk,m,s) =1*, but F_{SIG} is agnostic of its algorithms, i.e., defined for *any* algorithms
  - Change F_{SIG} to be parameterize by algorithms (otherwise follow [Can05,KT13])

- Still, he statement *verify(pk,m,s) =1* does **not** imply that F_{SIG} would answer with 1
  - Non-trivial…. and is not what we want!

Part III — Getting theory and practice right

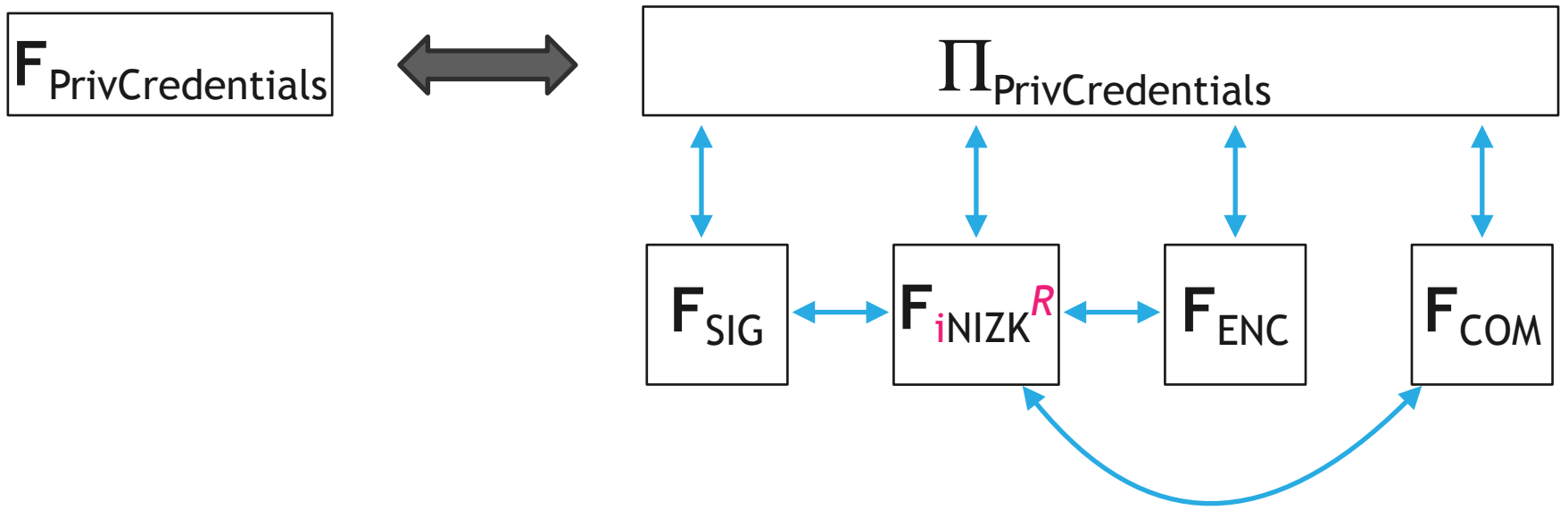
Modular ZK protocol usage:
What we want and how to do it

Let us UC-dream…. modular protocol construction



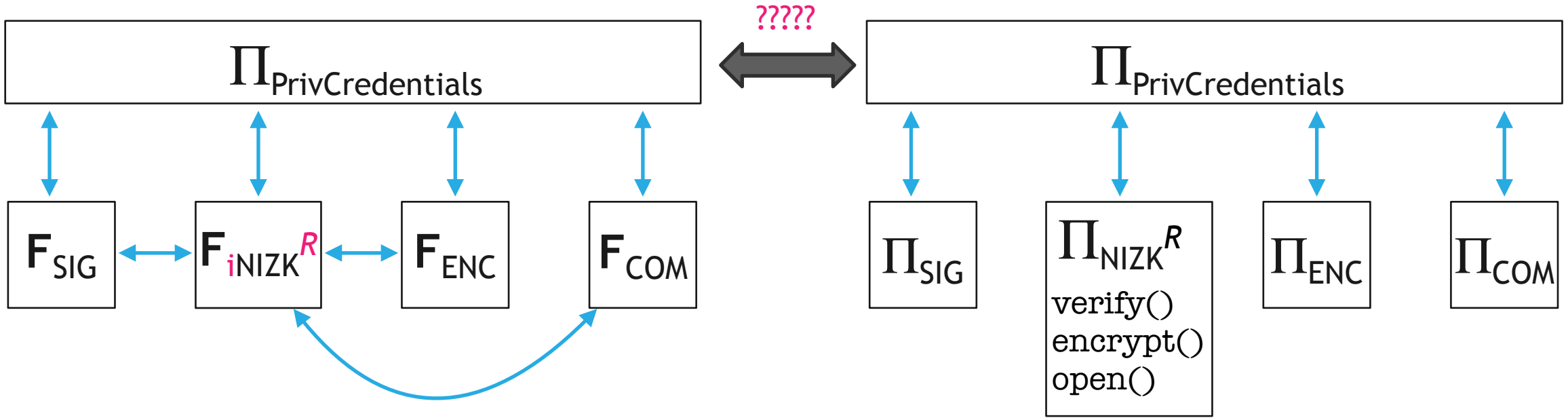
with relation R making statements about results of algorithms (e.g., verify, encrypt, and open) used in other functionality

Let us UC-dream…. modular protocol construction



with relation R making statements about results of **calls** to other functionalities!

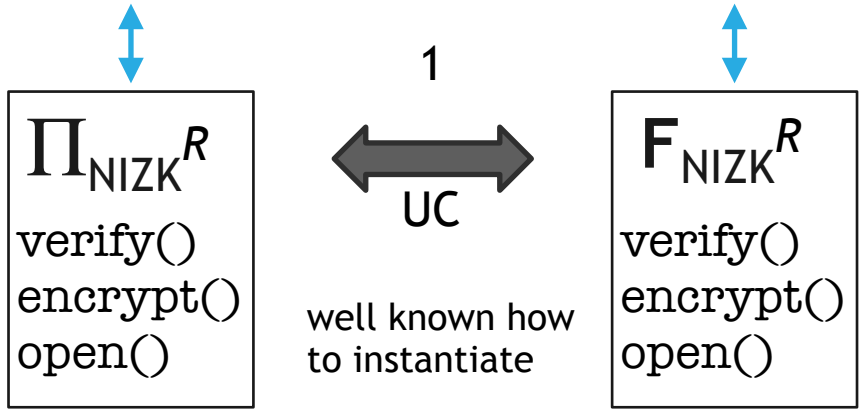
Let us UC-dream…. modular protocol construction



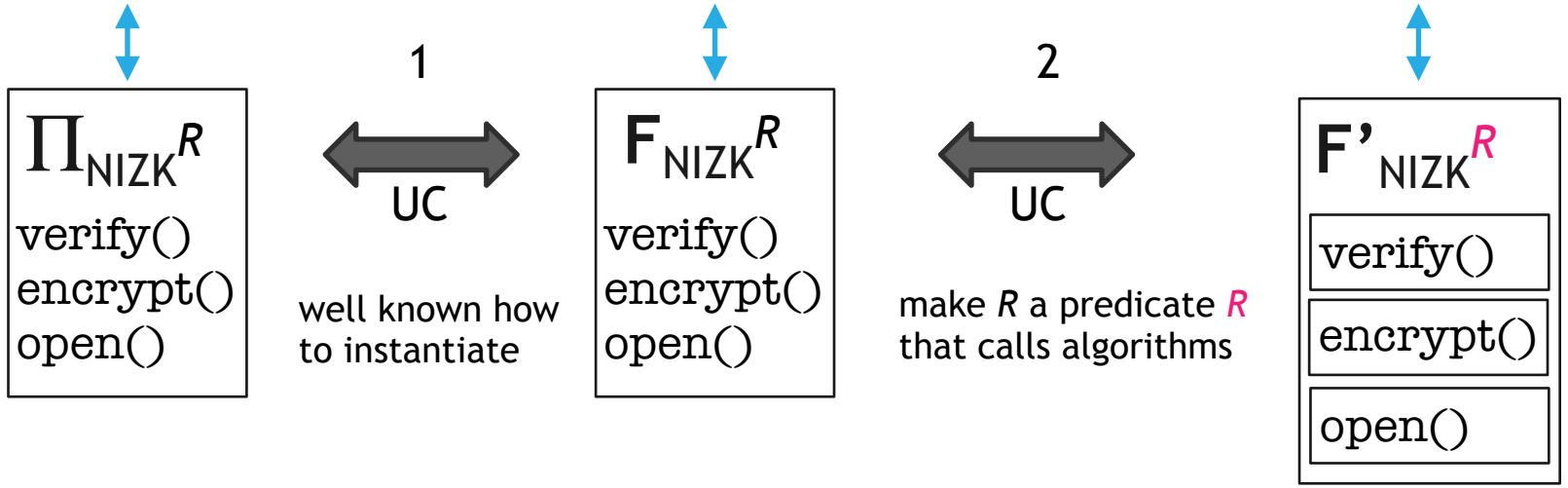
Two issues:

- $\Pi_{\text{PrivCredentials}}$ is not subroutine respective, i.e., UC theorem does not apply !
- Π_{NIZK}^{R} does not interact with other protocols and thus cannot realise F_{iNIZK}^{R} !

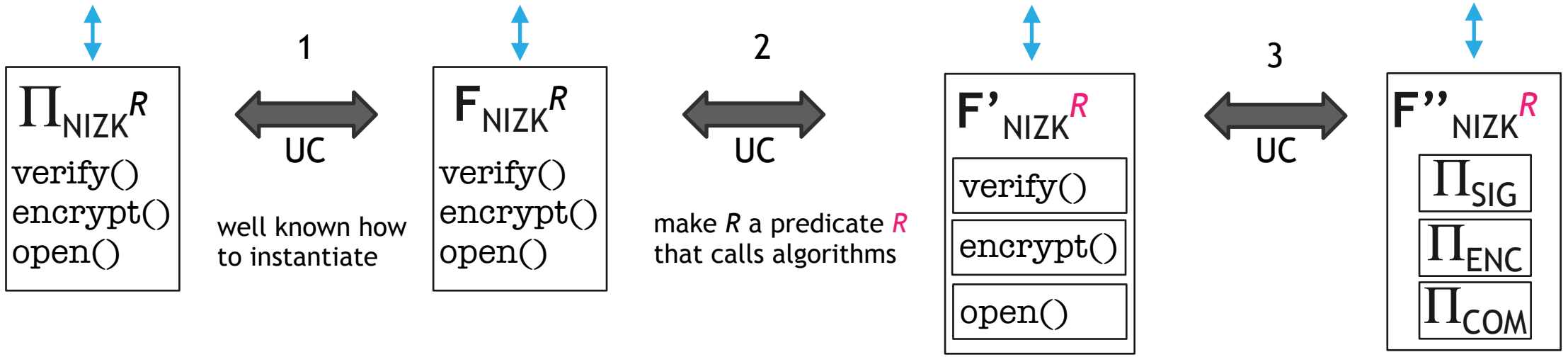
Composable modular protocol construction, finally!



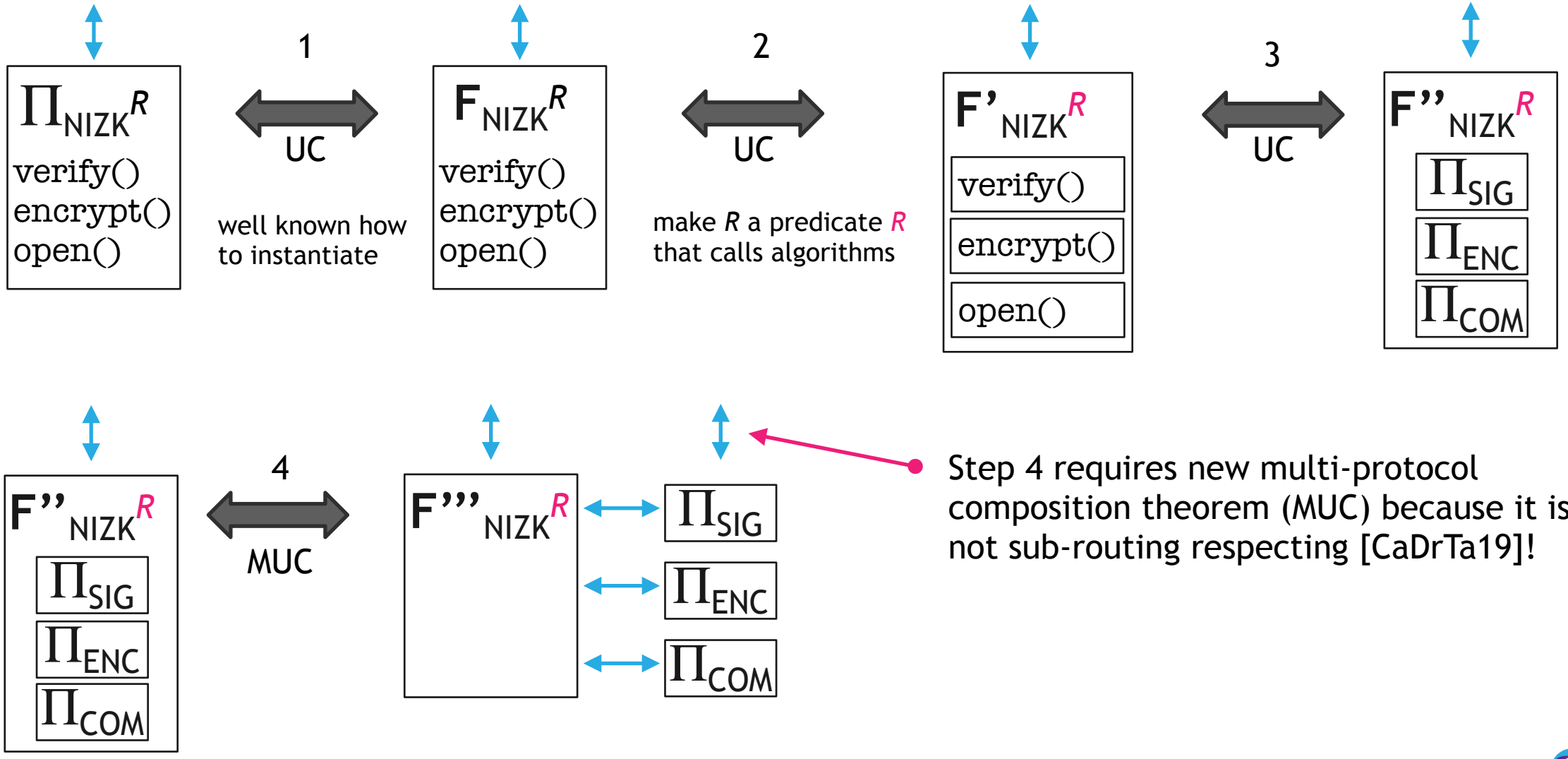
Composable modular protocol construction, finally!



Composable modular protocol construction, finally!

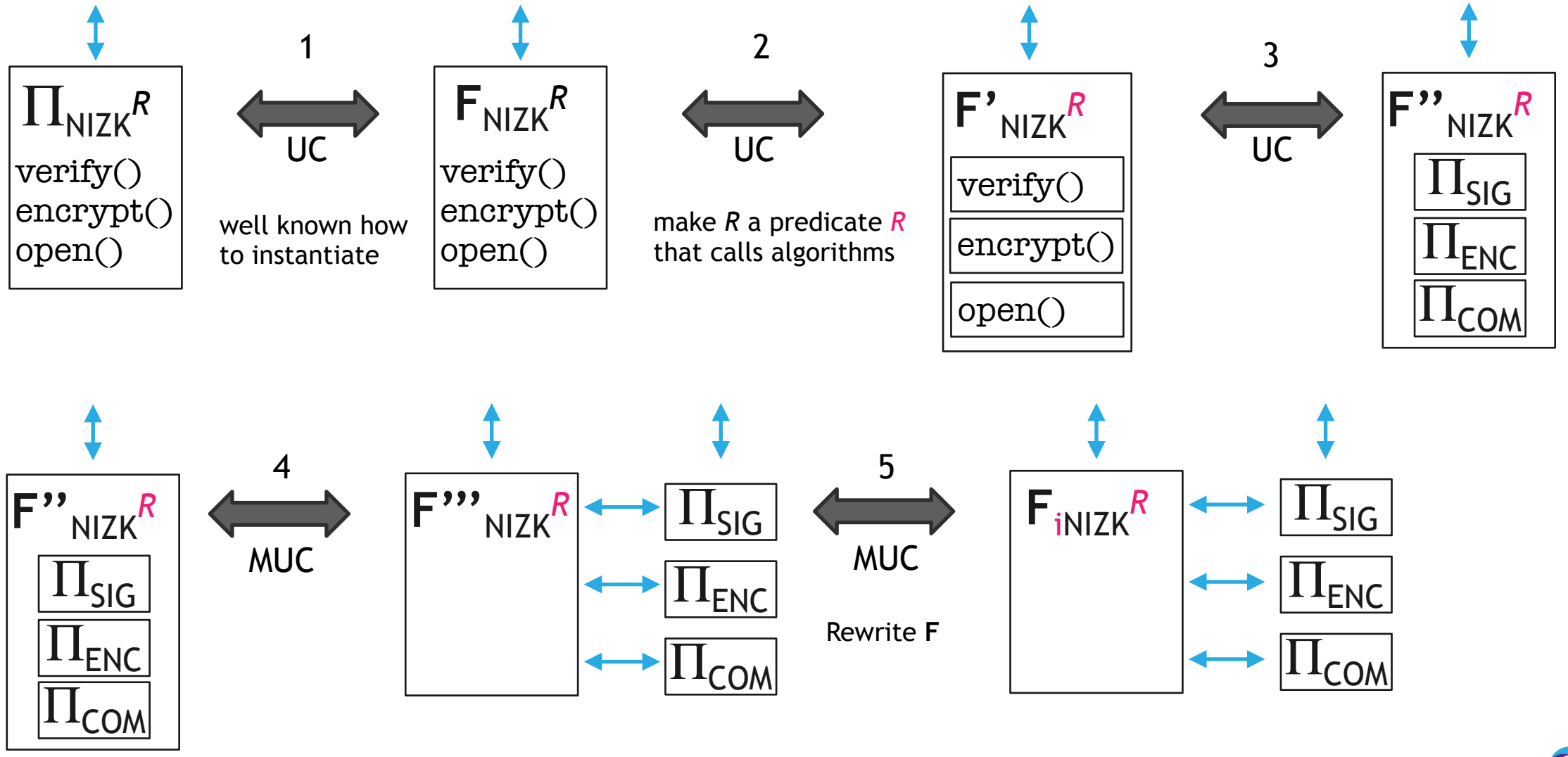


Composable modular protocol construction, finally!

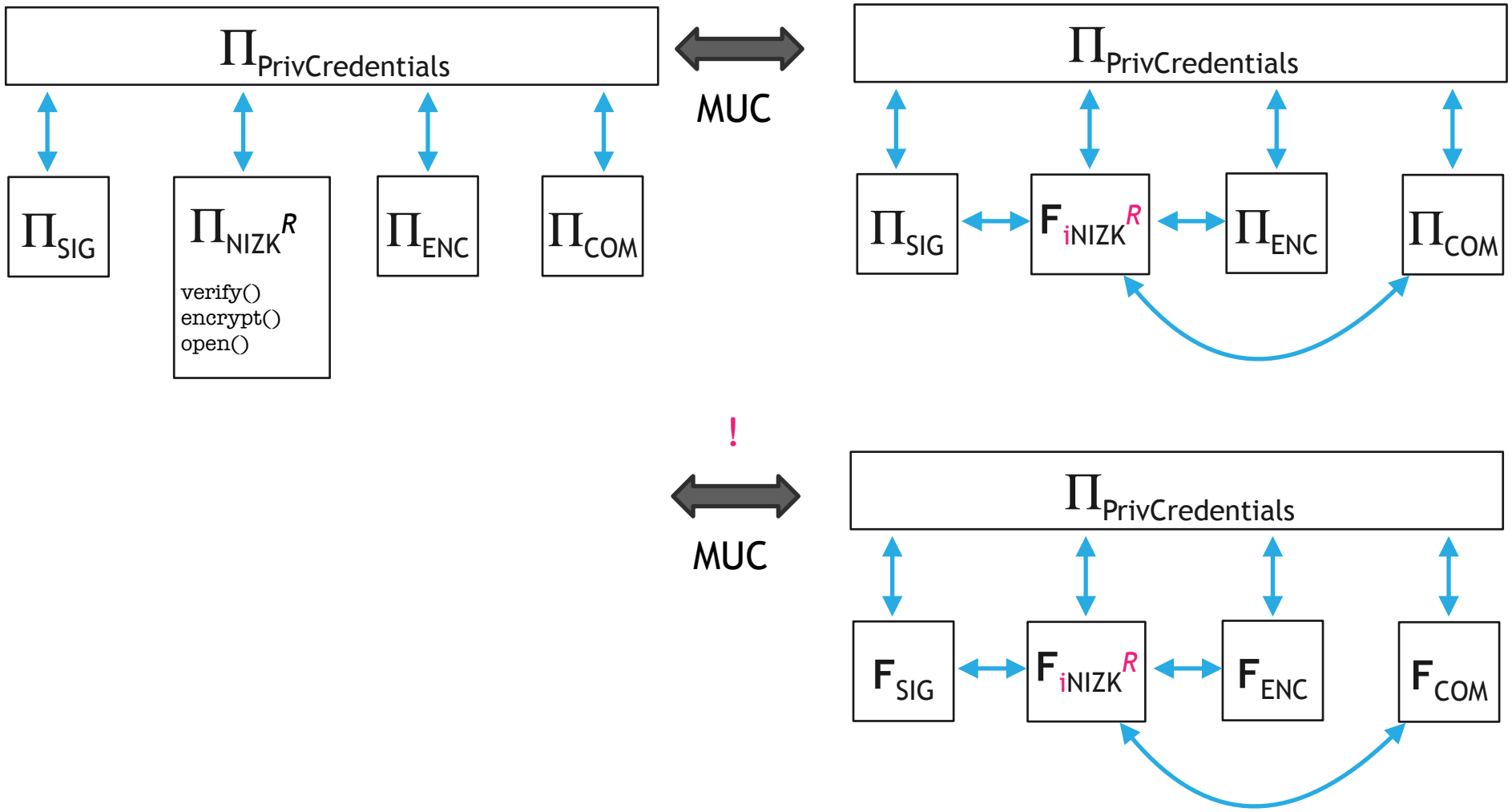


Step 4 requires new multi-protocol composition theorem (MUC) because it is not sub-routing respecting [CaDrTa19]!

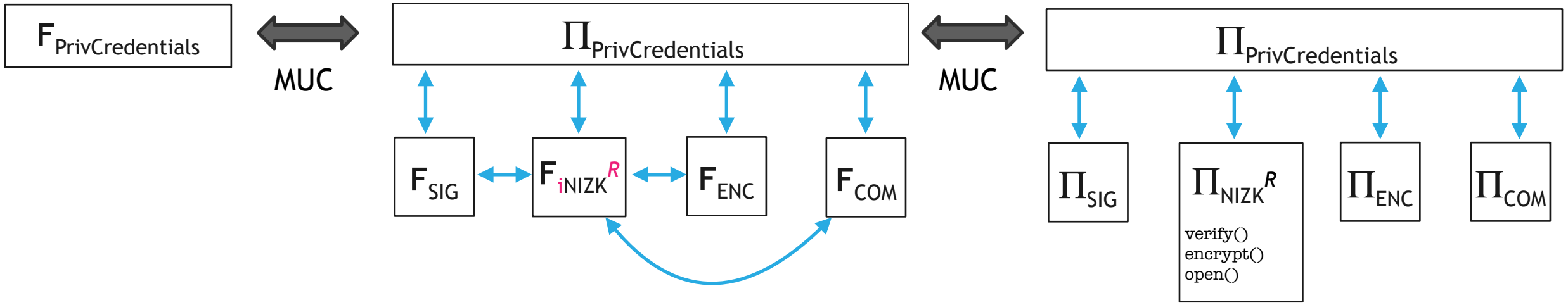
Composable modular protocol construction, finally!



Composable modular protocol construction, finally!



Composable modular protocol construction, finally!



Summary of proof method:

- proof needs to be done for each predicate *R*
- proof is generic, i.e., works for any instantiations, except Step 1 (realisation Π_{NIZK}^{R} of F_{NIZK}^{R})

Protocol design:

- 1) define overall F 2) define Π and F_{iNIZK}^{R} 3) prove security of Π 4) apply steps above!

Details see [Camenisch, Drijvers, Tackmann '19] https://ia.cr/2019/065 (also shows concrete examples)

Conclusion

Part I

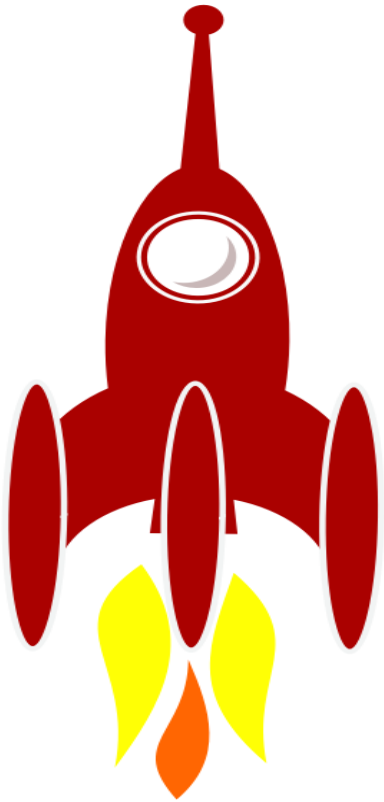
- A number of ZKP frameworks
- Generalised Schnorr proofs ready to be standardised
- More work is needed for post-quantum frameworks

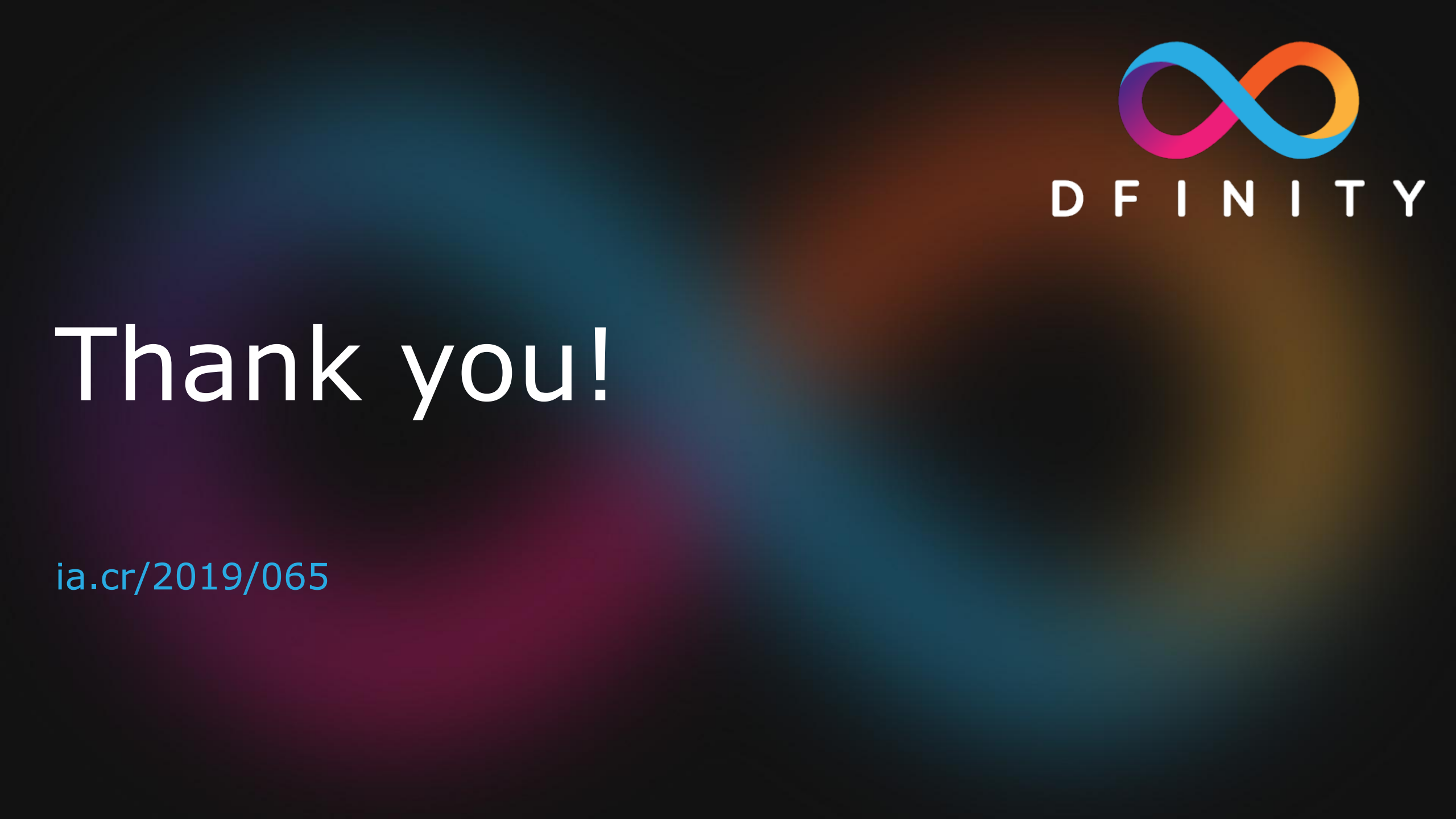
Part II

- The UC world is not as we would think it is…

Part III

- Modular proofs are very important!
- UC Functionalities in the literature are typically not suited for composability
- UC Theorem is too limited -> MUC Theorem
- New proof methodology for modular zero-knowledge functionality
  - Needs to be done per relation
  - But otherwise works for any realisation of functionality
- More work is needed before we can become space farers!

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Thank you!
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